The Amplitude Nth-Power Squeezing of Radiation Fields in the Degenerate Raman Process

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Abstract

In this paper we study the amplitude Nth-power squeezing of radiation fields in the degenerate Raman process by using the modified effective Hamiltonian approach recently suggested by us. We found that if the field is initially in a coherent state it will not get squeezing for any Nth-power; if the field is initially in a squeezed vacuum, it may get Nth-power squeezing. The time evolution of the field fluctuation was discussed. Its dependences on power-order N, mean photon number $\bar{n}$, and squeezing angle ξ are analyzed.

1 Introduction

Squeezed states of radiation fields have been studied considerably in recent years. Besides the normal squeezing[1] it is also possible to define higher-order squeezing. Hong and Mandel[2] defined the 2Nth-order squeezing, and Hillery[3] introduced the amplitude squared squeezing. More recently, Zhang et al[4] suggested the amplitude Nth-power squeezing(ANPS), which includes the normal squeezing and the amplituide-squared squeezing as special cases. All these higher-order squeezing have been shown to be independent nonclassical features of radiation fields[5]. ANPS of radiation fields has been studied in many quantum optics systems[4–12].

On the other hand, the degenerate Raman process(DRP) is one of the most interesting two-photon interactions between atoms and radiation fields, and has been studied intensively[13–16]. Usually, this process was studied by the full microscopic Hamiltonian approach(FMHA)[13–14], and the effective Hamiltonian approach(EHA)[15]. Generally speaking, FMHA gives exact solution, but it may be too complicated to be used in some situations. Although EHA is simpler than FMHA, it loses a phase factor, it can not be used to deal with the quantities involving the off-diagonal elements of the density matrix. To overcome these shortages we have suggested a modified effective Hamiltonian approach(MEHA)[16].

In this paper we use MEHA to study ANPS of radiation fields in DRP.

2 The Degenerate Raman Process(DRP)

The DRP refers to the interaction between a Λ-type three level atoms and a single mode of a radiation field(Fig.1).

The modified effective Hamiltonian for DRP is [17]

$$H_{MEH} = H_{EH} + H_S \tag{1}$$

$$H_{EH} = \lambda a^+ a(|e><g| + |g><e|) \tag{2}$$

is the effective Hamiltonian(when the detuning is very large, one can eliminate the upper level adiabatically and obtain it) and

$$H_S = -a^+ a(\beta_1 |g><g| + \beta_2 |e><e|) \tag{3}$$

is the part representing the ac Stark shift of atomic levels. β_1 and β_2 are the Stark parameters for levels $|g>$ and $|e>$, respectively.

If the initial state for the atom-field system is

$$|\Psi(0)\rangle = \sum_{n=0}^{\infty} q_n [C_g(0)|g,n\rangle + C_e(0)|e,n\rangle] \tag{4}$$

we can express the state for a later time as

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} q_n [C_g^n(t)|g,n\rangle + C_e^n(t)|e,n\rangle] \tag{5}$$

¿From the time-dependent Schrödinger equation we can obtain $C_g^n(t)$ and $C_e^n(t)$.

The reduced density matrix for the field can be expressed as

$$\rho(t) = \sum_{n,n'=0}^{\infty} \rho_{nn'}(t)|n\rangle\langle n'| \tag{6}$$

$$\rho_{nn'}(t) = q_n q_{n'}^* [C_g^n(t)C_g^{n'*}(t) + C_e^n(t)C_e^{n'*}(t)] \tag{7}$$

Supposing initially the atom is in the state $|g\rangle$, i.e. $C_g(0) = 1$, and $C_e(0) = 0$, and let $g_1 = g_2 = g$ for simplisity, we get

$$\rho_{nn'}(T) = q_n q_{n'}^* exp[-i(n-n')T]cos(n-n')T \tag{8}$$

in which $T = \lambda t$. We see that the diagonal elements ρ_{nn} are independent of time and just the photon distribution function of initial field.

3 The Amplitude Nth-Power Squeezing(ANPS)

The amplitude Nth-power squeezing of a radiation field is defined in terms of the following quantities[4]

$$Z_1(N) = \frac{1}{2}(a^N + a^{+N}), \qquad Z_2(N) = \frac{1}{2i}(a^N - a^{+N}) \tag{9}$$

$Z_1(N)$ and $Z_2(N)$ satisfy the commutation relation and the uncertainty relation

$$[Z_1(N), Z_2(N)] = \frac{i}{2}[a^N, a^{+N}] \tag{10}$$

$$\langle(\Delta Z_1(N))^2\rangle\langle(\Delta Z_2(N))^2\rangle \geq \frac{1}{16}|\langle[a^N, a^{+N}]\rangle|^2 \tag{11}$$

The field is said to be Nth-power squeezed if

$$\langle(\Delta Z_i(N))^2\rangle < \frac{1}{4}\langle[a^N, a^{+N}]\rangle \qquad (i = 1,2) \tag{12}$$

Here we introduce a parameter named squeezed degree $S_i(N)$

$$S_i(N) = \frac{D_i(N)}{C(N)} \qquad (i = 1,2) \tag{13}$$

where $C(N)$ and $D_i(N)$ are defined as

$$C(N) = \langle[a^N, a^{+N}]\rangle, \qquad D_i(N) = 4\langle(\Delta Z_i(N))^2\rangle - \langle[a^N, a^{+N}]\rangle \tag{14}$$

Then the field is Nth-power squeezed if $D_i(N) < 0$, ($S_i(N) < 0$). $S_i(N) = -1$ corresponds to 100% squeezing. In the following section we will study ANPS in DRP. We will consider several kinds of initial field states.

4 ANPS in DRP

4.1. For an Initial Coherent State

$$|\alpha\rangle = \sum_{n=0}^{\infty} q_n^c |n\rangle, \qquad \alpha = \bar{n}^{\frac{1}{2}} e^{i\xi_c}$$

$$q_n^c = Q_n^c e^{in\xi_c}, \qquad Q_n^c = (e^{-\bar{n}} \frac{\bar{n}^n}{n!})^{\frac{1}{2}} \tag{15}$$

then we have

$$\rho_{nn'}^c(T) = Q_n^c Q_{n'}^c exp[-i(n-n')(T-\xi_c)] cos(n-n')T \tag{16}$$

We can find

$$D_1(N) = 4\bar{n}^N sin^2(NT) sin^2[N(T-\xi_c)]$$

$$D_2(N) = 4\bar{n}^N sin^2(NT) cos^2[N(T-\xi_c)] \tag{17}$$

We see that in a degenerate Raman process the field will not get Nth-power squeezing if it is initially in a coherent state.

4.2. For an Initial Squeezed Vacuum

$$|0_{sq}\rangle = \sum_{n=0}^{\infty} q_{2n} |2n\rangle, \qquad q_{2n} = Q_{2n} e^{in\xi}$$

$$Q_{2n} = (\frac{1}{\bar{n}+1})^{\frac{1}{4}} [-\frac{1}{2}(\frac{\bar{n}}{\bar{n}+1})^{\frac{1}{2}})]^n \frac{[(2n)!]^{\frac{1}{2}}}{n!} \tag{18}$$

where $\bar{n}$ is the mean photon number and ξ is the squeezing angle of the initial field. Then we have

$$\rho_{2n,2n'}(T) = Q_{2n} Q_{2n'} exp[-i(n-n')(2T-\xi)] cos(n-n')2T \tag{19}$$

We see that only even-photon-number states can be found in a squeezed vacuum. The photon-number distribution function is

$$P_{2n} = \rho_{2n,2n} = Q_{2n} Q_{2n} \tag{20}$$

For $N = odd = 2M-1 (M = 1, 2, 3, ...)$ we can find

$$C(1) = 1$$

$$D_1(1) = 2\{\bar{n} - [\bar{n}(\bar{n}+1)]^{\frac{1}{2}} cos(2T-\xi) cos(2T)\}$$

$$D_2(1) = 2\{\bar{n} + [\bar{n}(\bar{n}+1)]^{\frac{1}{2}} cos(2T-\xi) cos(2T)\} \tag{21}$$

$$C(3) = 3(9\bar{n}^2 + 9\bar{n} + 2)$$

$$D_1(3) = 6\{\bar{n}^2(5\bar{n}+3) - 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}} cos(6T-3\xi) cos(6T)\}$$

$$D_2(3) = 6\{\bar{n}^2(5\bar{n}+3) + 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}} cos(6T-3\xi) cos(6T)\} \tag{22}$$

We can show that $[D_2(2M-1)]_{\xi=\pi} = [D_1(2M-1)]_{\xi=0}$ can be smaller than zero, but $[D_1(2M-1)]_{\xi=\pi} = [D_2(2M-1)]_{\xi=0}$ can not be smaller than zero. This shows that we can have squeezing in $Z_1(2M-1)$ components for $\xi = 0$ and in $Z_2(2M-1)$ components for $\xi = \pi$, but we have not squeezing in $Z_1(2M-1)$ components for $\xi = \pi$ and in $Z_2(2M-1)$ components for $\xi = 0$.

For $N = even = 2M (M = 1, 2, 3, ...)$ we have

$$C(2) = 2(2\bar{n} + 1)$$

$$D_1(2) = 2\bar{n}\{(3\bar{n}+1) + (\bar{n}+1)[3cos(4T-2\xi)cos(4T) - 2cos^2(2T-\xi)cos^2(2T)]\}$$
$$D_2(2) = 2\bar{n}\{(3\bar{n}+1) - (\bar{n}+1)[3cos(4T-2\xi)cos(4T) + 2sin^2(2T-\xi)cos^2(2T)]\} \tag{23}$$
$$C(4) = 24(10\bar{n}^3 + 15\bar{n}^2 + 7\bar{n} + 1)$$
$$D_1(4) = 6\bar{n}^2\{(35\bar{n}^2 + 30\bar{n} + 3) + (\bar{n}+1)^2[35cos(8T-4\xi) - 6cos^2(4T-2\xi)cos^2(4T)]\}$$
$$D_2(4) = 6\bar{n}^2\{(35\bar{n}^2 + 30\bar{n} + 3) - (\bar{n}+1)^2[35cos(8T-4\xi) + 6sin^2(4T-2\xi)cos^2(4T)]\} \tag{24}$$

We can show that $[D_2(2M)]_{\xi=\pi} = [D_2(2M)]_{\xi=0}$ can be smaller than zero, but $[D_1(2M)]_{\xi=\pi} = [D_1(2M)]_{\xi=0}$ can not be smaller than zero. This shows that we can have squeezing in $Z_2(2M)$ components for both $\xi = 0$ and $\xi = \pi$, but we can not get squeezing in $Z_1(2M)$ components for $\xi = 0$ and $\xi = \pi$.

We are also interested in the optimal squeezing.

$$[S(1)]_{min} = 2\{\bar{n} - [\bar{n}(\bar{n}+1)]^{\frac{1}{2}}\}$$
$$[S(2)]_{min} = -\frac{2\bar{n}}{2\bar{n}+1}$$
$$[S(3)]_{min} = \frac{2\{\bar{n}^2(5\bar{n}+3) - 5[\bar{n}(\bar{n}+1)]^{\frac{3}{2}}\}}{9\bar{n}(\bar{n}+1)+2}$$
$$[S(4)]_{min} = -\frac{2\bar{n}^2(5\bar{n}+4)}{10\bar{n}^3 + 15\bar{n}^2 + 7\bar{n} + 1} \tag{25}$$

We see that $[S(N)]_{min} \to 0$ when $\bar{n} \ll 1$, and $[S(N)]_{min} \to -1$ (100% squeezing) when $\bar{n} \gg 1$.

To see the features of the field fluctuation more clearly, we have done numerical calculation and drawn some figures(Fig.2-10). From these figures we see the follows:

1. Generally, the field fluctuation oscillates periodically, and the oscillation frequency is proportional to N(Fig.2-9).
2. For a given $\bar{n}$, the oscillation amplitude decreases as N increase (Fig.2-5).
3. For a given N, the oscillation amplitude increases as $\bar{n}$ increases, but $[S(N)]_{min}$ changes smaller as $\bar{n}$ increases (Fig.6-9). $S_{min} \to -1$ when $\bar{n} \gg 1$(Fig.10).

5 Conclusion

In this paper we have studied ANPS of radiation fields in DRP by using MEHA. We found that if the field is initially in a coherent state it will not get squeezing in any Nth-power; if the field is initially in a squeezed vacuum, it may get Nth-power squeezing. The relations between the time evolution of the field fluctuation with N, $\bar{n}$, and ξ are discussed.

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Figiure Captions

Fig.1 Schematic diagram of the degenerate Λ-type three-level atom interaction with a single-mode field. ω: frequency of field; δ: atom field detuning.
Fig.2 S_1 vs T. $\bar{n}$=0.1 a: N=1; b: N=3
Fig.3 S_1 vs T. $\bar{n}$=1.0 a: N=1; b: N=3
Fig.4 S_2 vs T. $\bar{n}$=0.1 a: N=2; b: N=4
Fig.5 S_2 vs T. $\bar{n}$=1.0 a: N=2; b: N=4
Fig.6 $S_1(1)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n}$ = 1.0; c: $\bar{n}$ = 5.0
Fig.7 $S_2(2)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n}$ = 1.0; c: $\bar{n}$ = 5.0
Fig.8 $S_1(3)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n}$ = 1.0; c: $\bar{n}$ = 5.0
Fig.9 $S_2(4)$ vs T. a: $\bar{n}$=0.1; b: $\bar{n}$ = 1.0; c: $\bar{n}$ = 5.0
Fig.10 $[S(N)]_{min}$ vs $\bar{n}$. a,b,c,d corresponde to N=1,2,3,4 espectively.

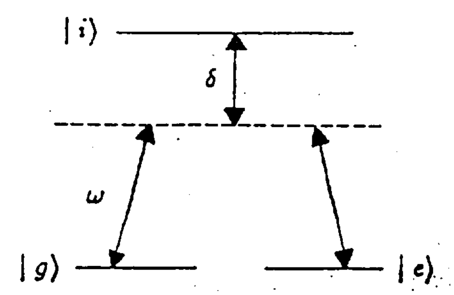

Fig.1

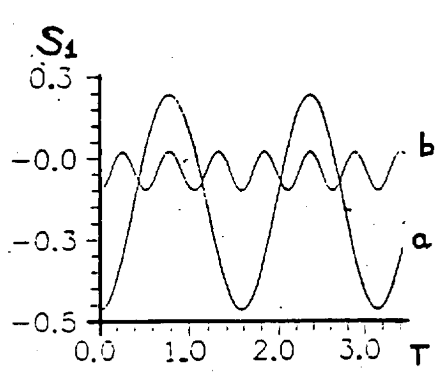

Fig.2

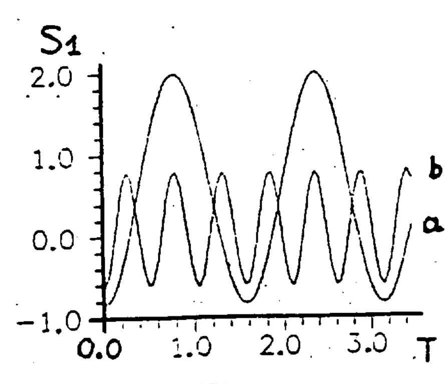

Fig.3

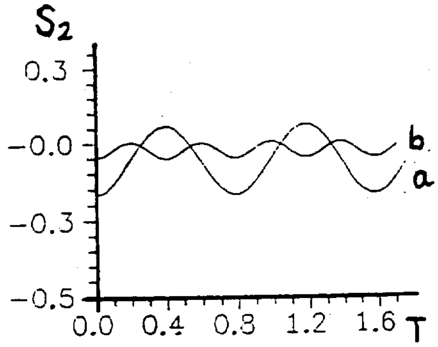

Fig.4

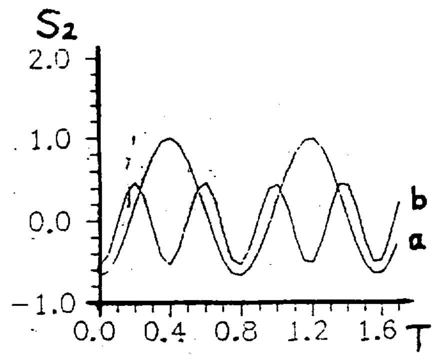

Fig.5

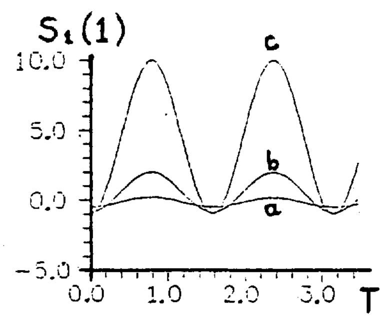

Fig.6

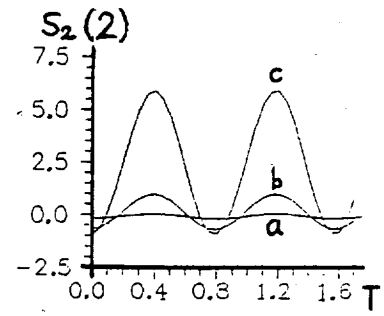

Fig.7

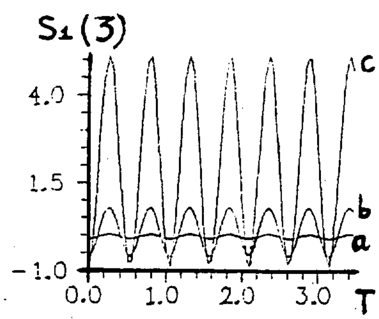

Fig.8

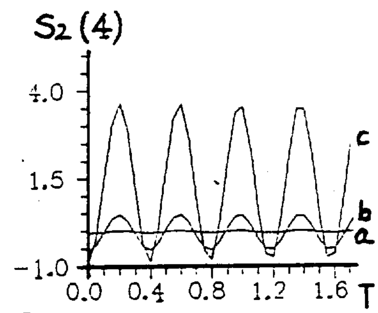

Fig.9

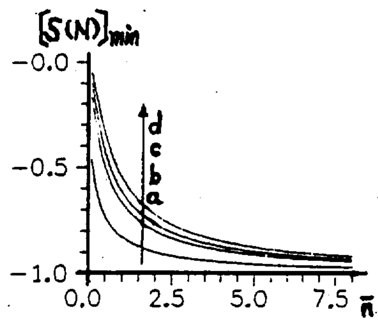

Fig.10